Q. $h[n] = 5^n u[n]$

Ans. option (d) is correct.

Q. if $H(Z) = \frac{1}{(1+\frac{1}{2}Z^{-1})(1-\frac{1}{4}Z^{-1})}$

ES-03
15 Marks

Represent $H(Z)$ in

1. Direct form
2. parallel "
3. cascade "

Ans. 1) Direct form:-

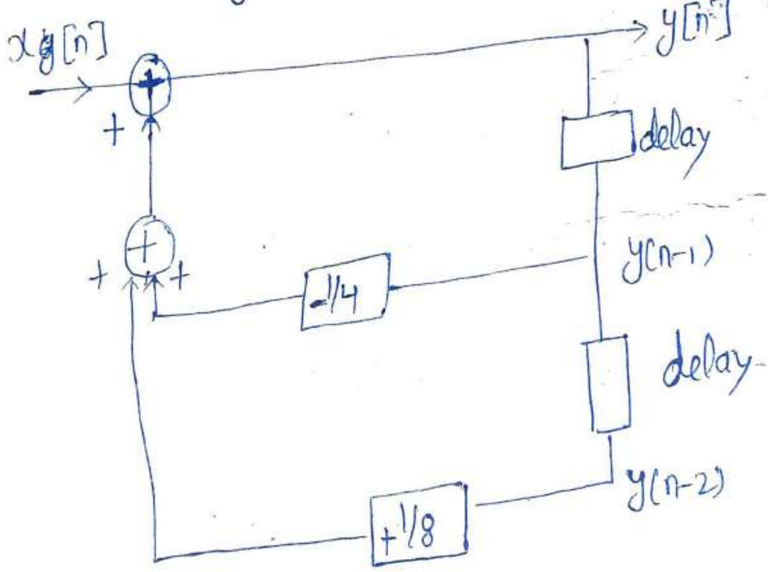
$$\frac{Y(Z)}{X(Z)} = \frac{1}{1-\frac{1}{4}Z^{-1}+\frac{1}{2}Z^{-1}-\frac{1}{8}Z^{-2}}$$

$$\frac{Y(Z)}{X(Z)} = \frac{1}{1+\frac{1}{4}Z^{-1}-\frac{1}{8}Z^{-2}}$$

$$Y(Z) + \frac{1}{4}Z^{-1}Y(Z) - \frac{1}{8}Z^{-2}Y(Z) = X(Z)$$

$$y[n] + \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n]$$

$$y[n] = x[n] + \frac{1}{8}y[n-2] - \frac{1}{4}y[n-1].$$



2) Parallel form :-

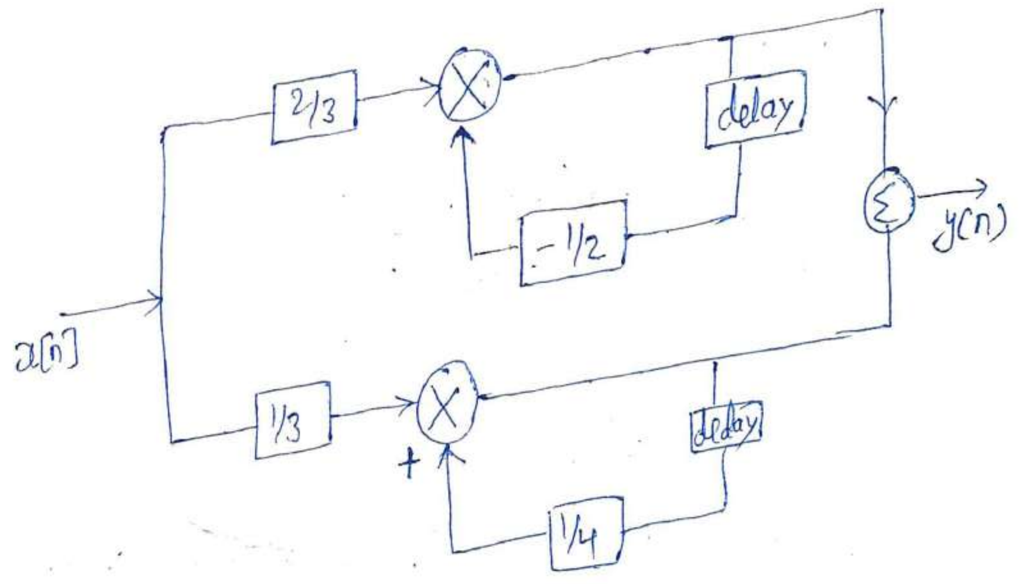
$$H(Z) = \frac{1}{(1+\frac{1}{2}Z^{-1})(1-\frac{1}{4}Z^{-1})}$$

Partial fraction.

$$\frac{Y(Z)}{X(Z)} = \frac{2/3}{1+\frac{1}{2}Z^{-1}} + \frac{1/3}{1-1/4 Z^{-1}}$$

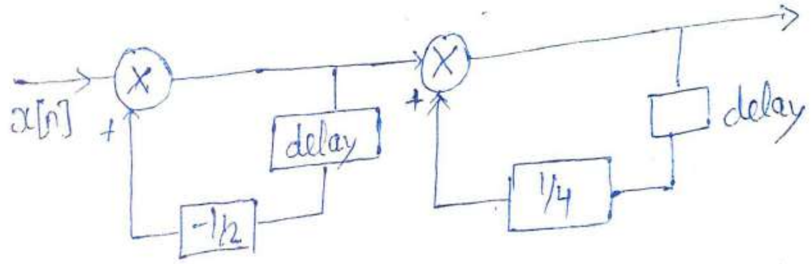
$$H(Z) = H_1(Z) + H_2(Z)$$

$$H_1(Z) = \frac{Y_1(Z)}{X_1(Z)} = \frac{2/3}{1+\frac{1}{2}Z^{-1}}$$



3) Cascade form →

$$H(Z) = \frac{1}{1+\frac{1}{2}Z^{-1}} \times \frac{1}{1-\frac{1}{4}Z^{-1}}$$



Initial & final value theorem →

$$x[0] = \lim_{z \to \infty} X(z) \rightarrow \text{Initial value theorem}$$

$$x[\infty] = \lim_{z \to 1} (1 - z^{-1}) X(z) \rightarrow \text{final value theorem.}$$

Q. If Z-T/F of a signal is given by

$$C(z) = \frac{z^{-1}(1 - z^{-4})}{4(1 - z^{-1})^2}$$

what is final value?

a) 0 B) 1/4

c) 1 D) ∞

Ans.

$$= \lim_{z \to 1} \cancel{(1 - z^{-1})} \frac{z^{-1}(1 - z^{-4})}{4(1 - z^{-1})^{\cancel{2}}}$$

$$= \lim_{z \to 1} \frac{z^{-1} - z^{-5}}{4(1 - z^{-1})}$$

$$= \lim_{z \to 1} \frac{z^4 - 1}{4(z^5 - z^4)} = \begin{cases} 1 - z^{-4} = (1 - z^{-2})(1 + z^{-2}) \\ (1 - z^{-1})(1 + z^{-1})(1 + z^{-2}) \end{cases}$$

$$= \lim_{z \to 1} \frac{(1 \times z^{-1})}{4(1 - z^{-1})} \times (1 - z^{-1})(1 + z^{-1})(1 + z^{-2})$$

$$= \lim_{z \to 1} z^{-1} \times \frac{(1 + z^{-1})(1 + z^{-2})}{4}$$

$$= \frac{1 \times 2 \times 2}{4} = 1.$$

By L' Hospital Rule:

OR

$$\lim_{z \to 1} \frac{1}{4} \times \frac{0 + 4z^{-5}}{1} = \frac{1}{4} \times 4 = (1)$$

Q. If $H(Z) = \dfrac{Z^2+1}{(Z-0.5)(Z+0.5)}$

what is value of $h(0)$?

Ans.

$$h(0) = \lim_{Z\to\infty} \left(\frac{1+Z^{-2}}{(1-0.5Z^{-1})(1+0.5Z^{-1})}\right)$$

$$= 1$$

As pole lie in unit circle so it is stable system.

Q. $x[n] = (1/4)^n u[-n] = (4)^{-n} u[-n]$

Then what is Z T/F ?

A) $\dfrac{4Z}{Z-1}$, $|Z| > 1/4$ c) $\dfrac{1}{1-4Z}$ $|Z| > \dfrac{1}{4}$

B) $\dfrac{4Z}{4Z-1}$ $|Z| < 1/4$ d) $\dfrac{1}{1-4Z}$ $|Z| < \dfrac{1}{4}$ (✓)

Ans.

$$4^n u[n] = \frac{1}{1-4Z^{-1}}$$

$$4^{-n} u[-n] = \frac{1}{1-4Z}$$

Q. $x[n] = (2/3)^{|n|}$

A. $\dfrac{-5Z}{(2Z-3)(3Z-2)}$ $-3/2 < |Z| < -2/3$ C) $\dfrac{5Z}{(2Z-3)(2Z-2)}$ $\dfrac{3}{2} < |Z| < \dfrac{2}{3}$

B. $\dfrac{-5Z}{(2Z-3)(3Z-2)}$ $\dfrac{2}{3} < |Z| < 3/2$ (✓) d) $\dfrac{5Z}{(2Z-3)(3Z-2)}$ $\dfrac{-3}{2} < |Z| < -\dfrac{2}{3}$.

Ans. $x[n] = b^{|n|}$

ROC is

$b < |Z| < 1/b$ where $0 < b < 1$.

$$Z = \frac{1 \pm \sqrt{1+\frac{1}{2}}}{2} = \frac{1 \pm \sqrt{2}}{2}.$$

$Z_1 = 1.2, -0.2$

↳ lie outside of unit circle so unstable.

Note:-

1. for stable system all pole must lie inside unit circle
2. for min. phase all poles & zeros must lie inside the unit circle.

Q. If $H(Z) = \frac{4Z^{-1}}{(1-\frac{1}{4}Z^{-1})^2}$, $|Z| > 1/4$

then System is

1. Causal
2. Stable

A) 1 B) only stable (2).

C) 1,2 (✓) d) None

Ans. $h(n) = 4 \times (1/4)^n u(n)$

$= 4 \times (1/4)^{n-1} u(n-1)$

So, causal System, Stable

Q. $H(Z) = \frac{\frac{1}{4}Z^{-1}}{(1-\frac{1}{4}Z^{-1})^2}$, $|Z| > \frac{1}{4}$

Ans. $h[n] = (1/4)^n u(n)$, $\left(\frac{1}{4}\right)^n u(n) \longrightarrow \frac{1}{(1-\frac{1}{4}Z^{-1})}$

$n\left(\frac{1}{4}\right)^n u(n) \longrightarrow -Z \times \frac{d}{dZ}\left(\frac{1}{1-\frac{1}{4}Z^{-1}}\right)$

$h[n] = 16\, n\, (1/4)^n u[n].$

$n<0$, $h[n]=0$, System causal

Time Response of System:-

1) Zero - step → In this case I/p is zero & o/p is only due to initials condition

2) Zero - I/p → In this case initial cond's are zero & o/p is only due to I/p.

IES

Q. Transfer function is defined for

1. Zero step

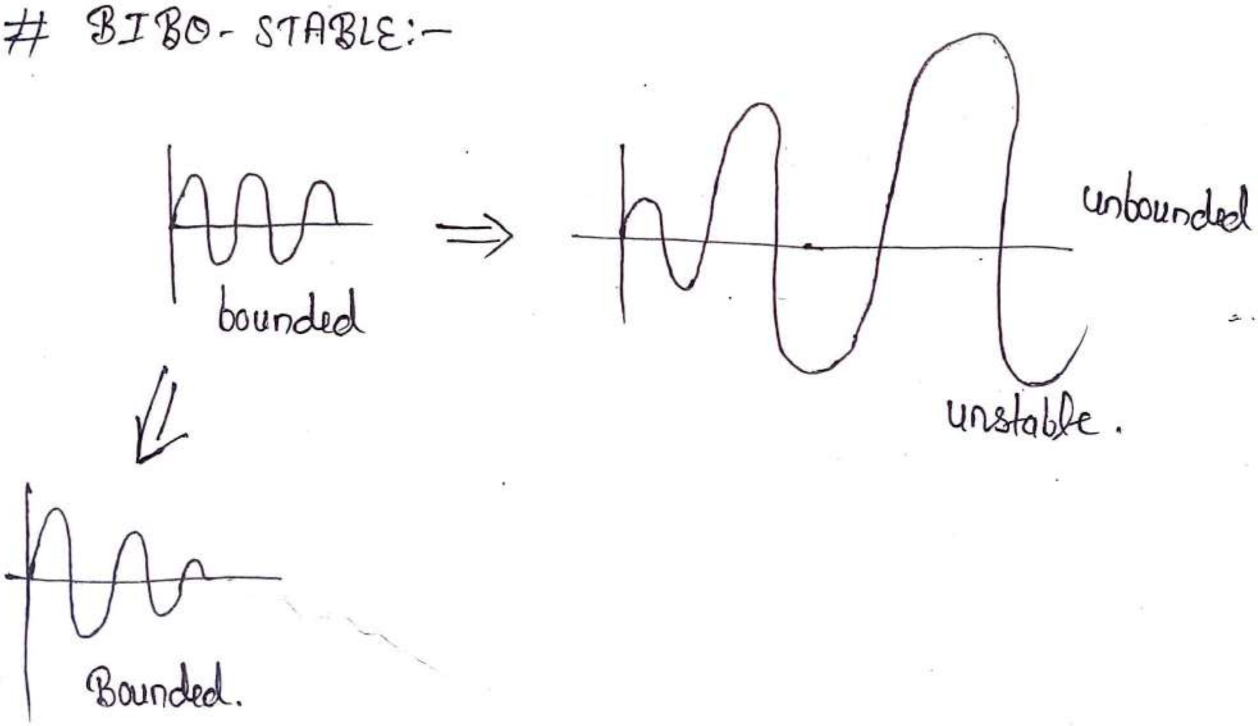
2) Zero I/p

a) 1 (✓)   c) 1, 2

b) 2   D) None

Total Response = Zero step + Zero I/p.

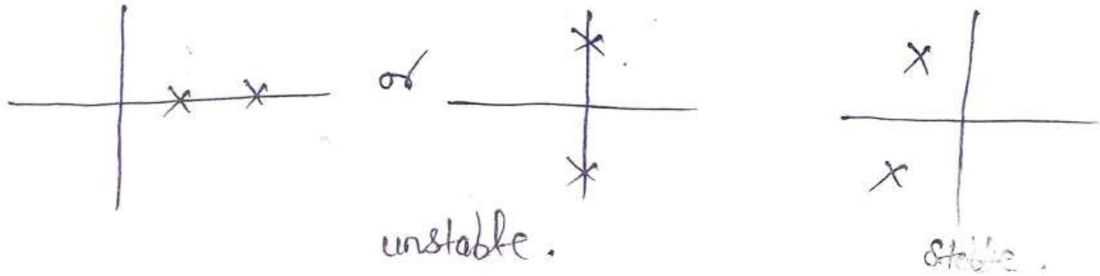
BIBO - STABLE:-



condition for BIBO - stable →

(i) Roots must be on left hand side i.e, -ve real axis.

(ii) If roots are on +ve real axis or on jω-axis then it is not stable system.



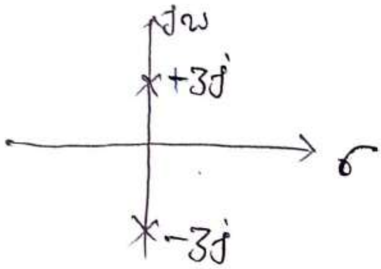
Asymptotic Stable or Zero o/p stable →

condition → 1. $|y(t)| < \infty$ } Con. for asymptotic stable

2. $\lim_{t \to \infty} |y(t)| = 0$

Location of Root's must be only on -ve real axis

If root's are on $j\omega$-axis then System is called as Marginally Stable or Marginally unstable system. i.e oscillation are present.



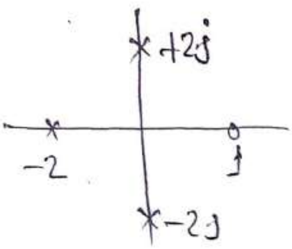
Q. $M(s) = \dfrac{20}{(s+1)(s+2)(s+3)}$ → Stable System

as root's are on -ve real axis.

Q. $M(s) = \dfrac{20(s-1)}{(s+2)(s^2+4)}$

a) stable c) Marginally stable

b) unstable d) Marginally stable or unstable (✓)



Q.

$M(s) = \dfrac{20(s+1)}{(s-1)(s^2+2s+2)}$

unstable System due to poles on +ve real axis.

Q. $M(s) = \frac{20}{(s^2+4)(s+10)}$

⇓

Marginally stable or unstable.

If multiple order roots are present on jω-axis then system will become unstable

Q. $M(s) = \frac{20}{(s^2+4)^2(s+10)}$

unstable

Q. $M(s) = \frac{20}{(s^2+2s)}$ or $\frac{20}{s(s+2)}$

If any pole is placed intentionally then system is stable.

Q. $M(s) = \frac{10}{s^4+30s^3+s^2+10s}$

⇓

Stable System.

Transient Response & Steady State Response →

$y(t) = \underbrace{1}_{\text{Steady-state}} + \overbrace{e^{-4t}}^{\text{Transient Response}}$

Transient Response :- It is defined as part of response that goes to zero as time become very large so it $\lim_{t\to\infty} y(t) = 0$ this is condition for transient response.

Steady-state :- It is part of total response that remains after transient has died out.

If $\lim_{t\to\infty} y(t) \neq 0$ is cond. for steady state.

Q. When time period of observation is large then type of error is

Sol.n
a) transient Error
b) steady state Error (✓)
c) Half-power Error
d) position Error const.

Q. Which response remain nature of response (oscill. or overdamp)

IES

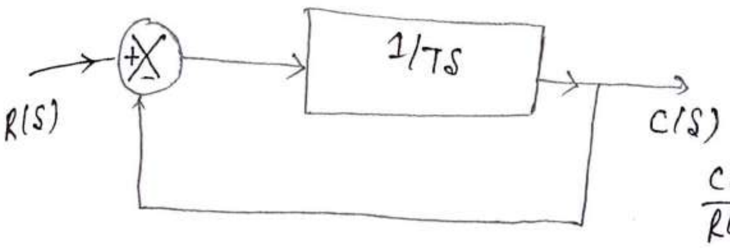
a) Transient (✓)
b) steady state
c) Half-Power
d) position

So, steady-state ~~talk~~ part remains accuracy of C.S.

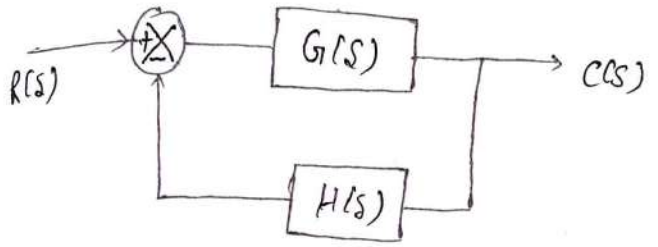
Transient	Steady-state
i) Time Response upto 3T or 4T	i) After 4T is called as steady state
ii) It doesn't depend on i/p signal	ii) It depend on i/p signal.
iii) It ~~remains~~ reveals nature of response & given indication about speed.	iii) It reveals accuracy.
iv) Natural response	iv) forced response.

Q. In case initial condition of a system are specified to be zero, it ~~emplo~~ implies that system is
A) working with zero ref. i/p.
B) works with Ref. i/p
C) at Rests but stores energy
d) at rest without stored energy. (✓)

Time-Response of 1st order System:-



$$\frac{C(s)}{R(s)} = \frac{1/TS}{1+1/TS} = \frac{1}{1+TS}$$



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

Case I:- If I/P is given as step fn.

$$r(t) = u(t)$$

$$\Rightarrow R(s) = 1/s$$

$$\frac{C(s)}{R(s)} = \frac{1}{1+TS}$$

$$C(s) = \frac{1}{S(1+TS)} = \frac{1}{S} - \frac{T}{1+TS}$$

$$\left[\because L e^{-at} = \frac{1}{s+a}\right]$$

$$= \frac{1}{s} - \frac{1}{s+1/T}$$

$$C(s) = \frac{1}{S} - \frac{1}{s+\frac{1}{T}}$$

$$c(t) = u(t) - e^{-t/T} u(t)$$

$$\boxed{c(t) = u(t)\left(1-e^{-t/T}\right)}$$

Case2 :- If i/p is given as Impulse :- $r(t) = \delta(t)$

$R(s) = 1$

$$\frac{C(s)}{R(s)} = \frac{1}{1+Ts}$$

$$C(s) = \frac{1}{(1+Ts)} R(s)$$

$$= \frac{1}{T(1/T+s)} = \frac{1}{T} e^{-(t/T)}$$

Case 3 → If i/p is Ramp $r(t) = t$

$$L(t^n) = \frac{\lfloor n}{s^{n+1}}$$

$$R(s) = \frac{1}{s^2}$$

$$C(s) = \frac{1}{s^2(1+Ts)} = \frac{A}{1+Ts} + \frac{B}{s} + \frac{C}{s^2}$$

$$C(s) = \frac{1}{s^2} - \frac{T}{s} + \frac{T^2}{1+sT}$$

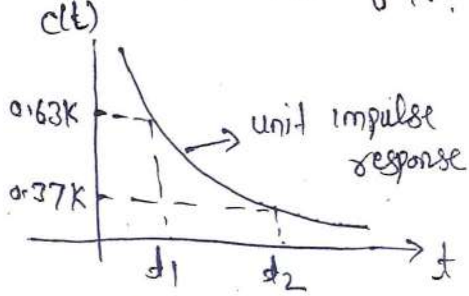
$$= \frac{1}{s^2} - \frac{T}{s} + \frac{T}{(1/T+s)}$$

$$C(t) = t\,u(t) - T u(t) + T e^{-t/T} u(t).$$

GATE-98

Q. unit impulse Response of a system having transfer f^n

$\frac{K}{s+a}$ as shown above what is value of K?



A) t_1 B) t_2

C) $1/t_1$ D) $1/t_2$ (✓)

Sol^n

o/p = (T/F) f^n × L(i/p)

$$C(s) = \frac{K}{s+a} \cdot 1 = \frac{K}{s+a}$$

$$c(t) = Ke^{-at}$$

$$= Ke^{-(t/t_1)}$$

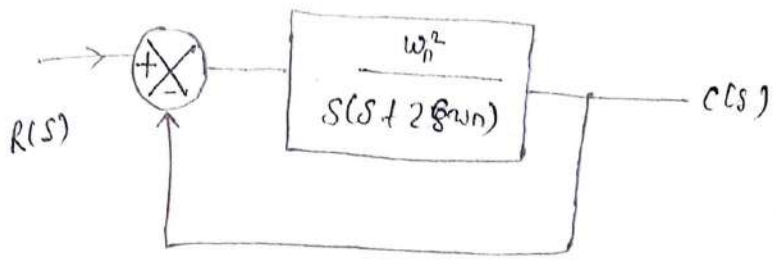
$$= ??$$

$$Ke^{-(t/t_2)}$$

$$Ke^{-1} = 0.37K$$

(4)

II^{nd} order System →



$$\frac{C(S)}{R(S)} = \frac{\frac{\omega_n^2}{S(S+2\xi\omega_n)}}{1+\frac{\omega_n^2}{S(S+2\xi\omega_n)}} = \frac{\omega_n^2}{S^2+2\xi S\omega_n+\omega_n^2}$$

$$= \frac{G(S)}{1+G(S)H(S)}$$

Note
So, for calculating values of ξ & ω_n always use characteristic Equ.

$$1+G(S)H(S)=0$$

Q. If $\frac{C(S)}{R(S)} = \frac{225}{S^2+30S+225}$ what is value of ξ & ω_n?

Sol.ⁿ

$$\frac{225}{S^2+30S+225} = \frac{G(S)}{1+G(S)H(S)}$$

$$S^2+2\xi\omega_n S+\omega_n^2$$

$$\omega_n^2 = 225 \Rightarrow \omega_n = 15$$

$$2\xi\omega_n = 30 \Rightarrow \xi = 1.$$

Q. $C(S) = \underbrace{\frac{400}{S^2+90S+900}}$

$$\Downarrow$$

$$S^2+2\xi\omega_n S+\omega_n^2$$

$$\omega_n = 30,\ \xi = 1.5$$

Q. If $G(s) = \frac{80}{s(s+18)}$

for unity feedback

$1+G(s)H(s)=0$

$1+\frac{80}{s(s+18)} \times 1 = 0$

$s^2+18s+80=0$

$\omega_n = \sqrt{80}$

$2\xi\omega_n = 18$

$2\xi \times \sqrt{80} = 18$

$\rightarrow \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

If $r(t)=u(t) \Rightarrow R(s) = 1/s$

$C(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$

$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t+\phi)$

$\Rightarrow \omega_d = \omega_n\sqrt{1-\xi^2}$

$\Rightarrow \cos\phi = \xi$

$\rightarrow \phi = \cos^{-1}\xi$

Characteristic equ. is

$s^2+2\xi\omega_n s+\omega_n^2=0$

$s = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2-4\omega_n^2}}{2}$

$s = \frac{-2\xi\omega_n \pm 2\sqrt{\omega_n^2-\xi^2\omega_n^2}}{2}$

$s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} \Rightarrow \boxed{s = -\xi\omega_n}$

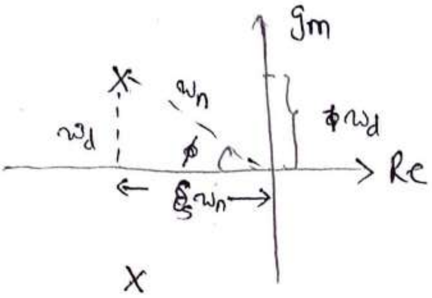
$s = -\alpha \pm j\omega_d$ (5)

where $\alpha = \xi\omega_n \rightarrow$ Damping factor

$\xi \rightarrow$ Damping ratio

time constant $= \frac{1}{\xi\omega_n}$

So, Inverse of damping factor is known as time constant.



$\boxed{\cos\phi = \xi}$

$\alpha > 0$, $\xi > 0$ 1. Positive Damping $\rightarrow$ Stable

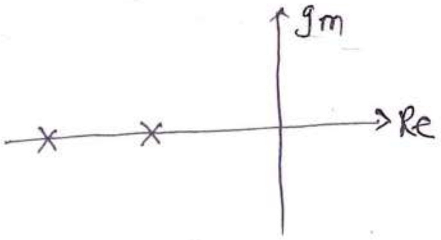
$\alpha < 0$, $\xi < 0$ 2. -ve Damping $\rightarrow$ unstable

$\alpha = 0$, $\xi = 0$ 3. Zero damping. $\rightarrow$ Marginally stable or unstable.

$\rightarrow \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$\rightarrow$ Determination of Step-Response for II-order control System

Case I :-

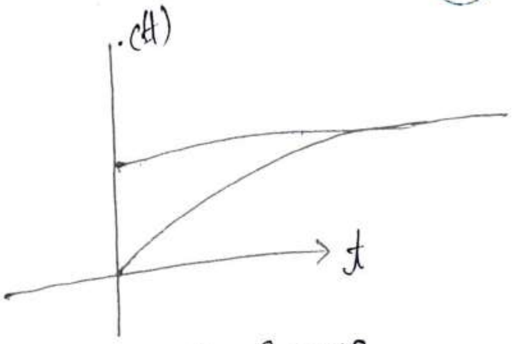


$\xi > 1$

i) under-damped $\xi < 1$

ii) over-damped $\xi > 1$

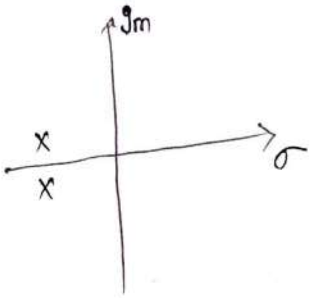
iii) critical " $\xi = 1$



Step Response

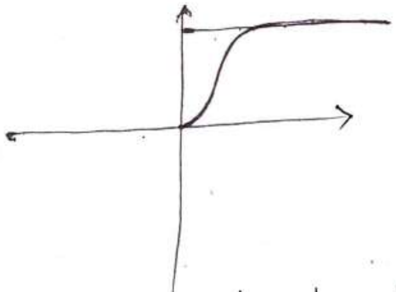
case 2:- critical Damped

Roots are repeated in nature

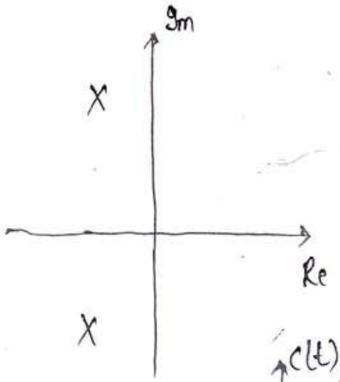


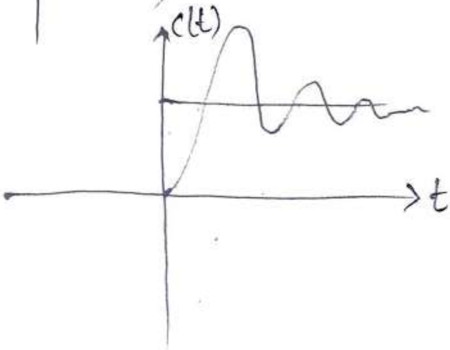
$$\frac{4}{s^2+4s+4}$$

$$(s+2)^2 = 0$$

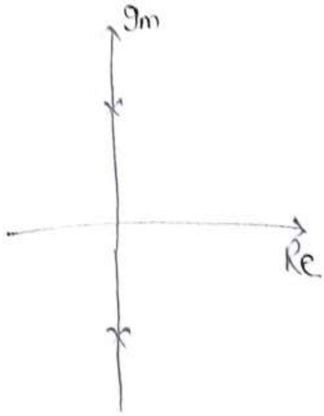


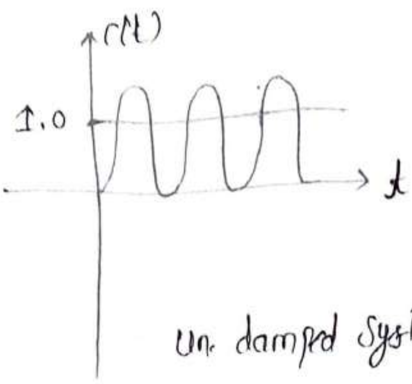
case 3:- $\xi < 1$ under-damped





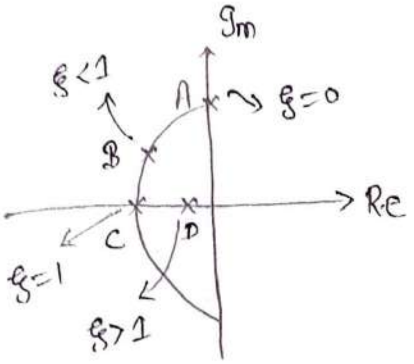
Case 4: $\xi = 0$





Un. damped system.

/08/07 for +ve & zero damping →



1. $\xi = 0$ Roots are imaginary
   → un damped

2. $\xi < 1$ Root's are complex
   → under-damped

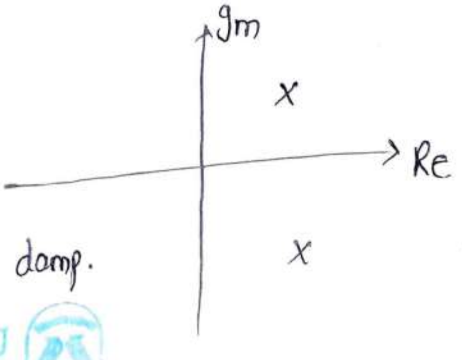
3. $\xi = 1$ → Roots are real & ~~complex~~. Equal
   → critical damped

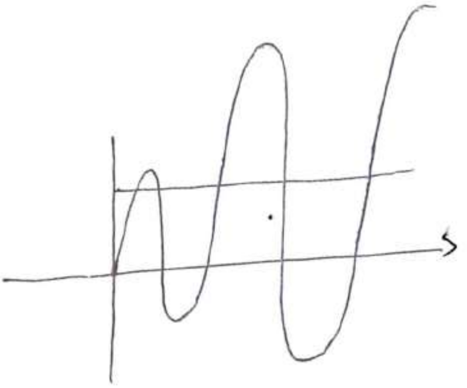
4. $\xi > 1$ → Roots are real & unequal
   → over-damping

'-ve' Damping →

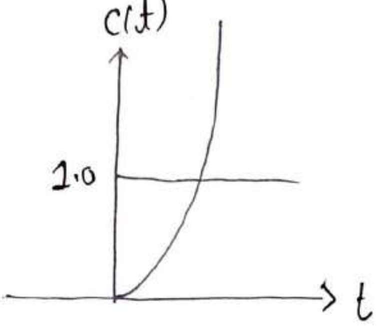
1. $-1 < \xi < 0$

Similar to under-damped system of +ve damp.





case 2:- $\xi < -1$ → Similar to over-damped system of positive damping.



Q. An under-damped sec. order system with -ve damping will have two roots
IES-05

A. one -ve real axis as real roots

B. Left hand side of complex plane as complex roots.

C. Right hand side of complex plane as complex conjugates. (✓)

D. on +ve real axis as real roots.

Q. for a II-order system damping ratio ξ is $0<\xi<1$ The roots of charact. polynomial are
IES 95

A) Real but not Equal

B) Real & Equal

C) complex Conjugate (✓)

D) Imaginary.

Q. The poles of a continuous time oscillations
GATE

A) Conjugate (✓)

B) Equal

D) Not Related

C) Zero

Time - Domain Specification →

i) Peak - Time
ii) Peak - overshoot
iii) Rise - time
iv) Delay Time
v) Settling Time

Peak - Time

o/p of unit step i/p to a sec. - order system is

$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$$

where,
$$\cos\phi = \xi$$
$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

for peak time $\frac{dy(t)}{dt} = 0$

$$\boxed{t_p = \frac{n\pi}{\omega_d}}$$

∴ Peak - Time

$$\boxed{t_p = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}}}$$

$n = 1, 2, 3, 4$ ---- so on.

If $n = 1, 3, 5, 7$ --- overshoot

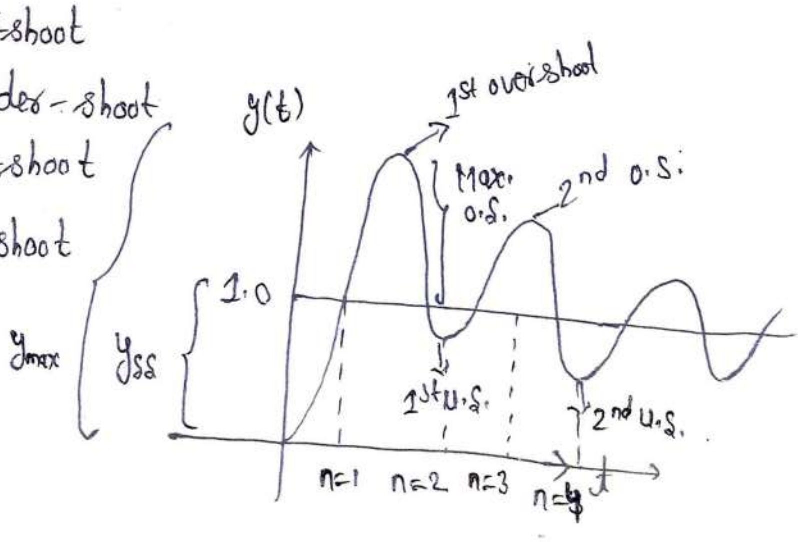
If $n = 2, 4, 6, 8$ ---- undershoot

$n = 1$ → 1st over-shoot

$n = 2$ → 1st under-shoot

$n = 3$ → IInd over-shoot

$n = 4$ → II - under-shoot



Max. O.S $= y_{max} - y_{ss}$

$$\% \text{ overshoot} = \frac{\text{Max. overshoot}}{y_{ss}}$$

$y_{ss} \rightarrow$ Steady-state

$$\% \text{ overshoot} = \frac{y_{max} - y_{ss}}{y_{ss}} \times 100\%$$

$$\% M_p = e^{-\pi \xi / \sqrt{1-\xi^2}} \times 100\%$$

$$\left.\begin{array}{l} y_{max} = y(t = t_p) \\ y_{ss} = 1 \end{array}\right\} \% M_p = \frac{y(t=t_p) - 1}{1} \times 100\%$$

for more stability values of peak overshoot must be as larger possible.

). IES-97, IES-06.

What value of K, for a unity feed back system with

$$G(s) = \frac{K}{s(1+s)}$$ to have a peak overshoot of 50%

a) 0.53

B) 5.3 (✓)

c) 0.6

d) 0.047

$$e^{\frac{\pi \xi}{\sqrt{1-\xi^2}}} = 2$$

$$\frac{\pi \xi}{\sqrt{1-\xi^2}} = \ln 2$$

$$\log_{10} 2 = 0.3010$$

$$\frac{\pi \xi}{\sqrt{1-\xi^2}} = 0.6932$$

$$\log_e 2 = \frac{\log_{10} 2}{\log_{10} e} = 2.303 \log_{10} 2$$

$$\therefore \quad \frac{\xi}{\sqrt{1-\xi^2}} = \frac{0.6932}{3.14} = 0.22$$

$$\frac{\xi}{\sqrt{1-\xi^2}} = 0.22 \Rightarrow \frac{\xi^2}{1-\xi^2} = 0.484$$

$$\xi^2 = 0.326 \simeq 0.33$$

$$\xi \approx 0.57$$

$$G(s) = \frac{K}{s(1+s)}$$

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(1+s)} = 0$$

$$s(1+s) + K = 0$$

$$s^2 + s + K = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n^2 = K \Rightarrow \omega_n = \sqrt{K}$$

$$2\xi\omega_n = 1$$

$$2\xi\sqrt{K} = 1$$

$$2 \times 0.57\sqrt{K} = 1$$

$$\sqrt{K} = \frac{1}{2 \times 0.57}$$

$$K = 0.76$$

$1 + G(s)H(s) = 0$

$$1 + \frac{1}{s(s+1)} = 0$$

$s(s+1) + 1 = 0$

$s^2 + s + 1 = 0$

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$\omega_n = 1, \quad 2\xi\omega_n = 1$

$2\xi = 1$

$\xi = 1/2 = 0.5$

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$= e^{-\pi \times 1/2/\sqrt{1-1/4}} = e^{\frac{-\pi/2}{\sqrt{3/4}}} = 0.163$$

M_p = Max overshoot − e_{ss}

Delay-time :- It is time required for response to reach 50% of final value in first attempt.

$$t_d = \frac{1 + 0.7\xi}{\omega_n} \quad \boxed{0 < \xi < 1.0}$$

$$\left.\begin{array}{l} t_d \propto \xi \\ t_d \propto \frac{1}{\omega_n} \end{array}\right\}$$

Rise-time :-

0 − 100% of its final value for under damped.

10% − 90% of its final value for over-damped.

$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$$

for calculation of Rise time simply put

$y(t) = 1$

$\sin(\omega_d t_r + \phi) = 0$

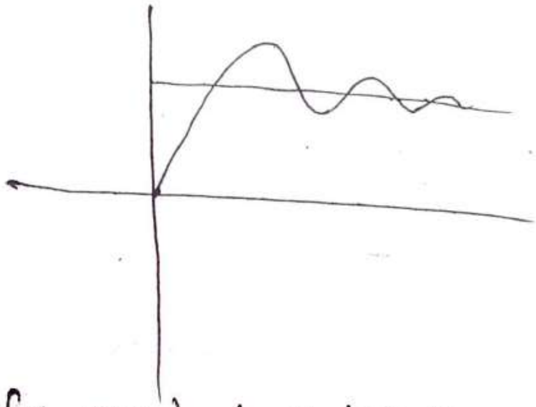
$$\omega_d t_r + \phi = \pi$$

$$\boxed{t_r = \frac{\pi - \phi}{\omega_d}}$$

$$t_r = \frac{0.8 + 2.5\xi}{\omega_n}$$

$$0 < \xi < 1.$$

Settling time → It is time required by system to reach & stay within a specified tolerance band (2% or 5%) of its final value.



for 2% band it takes 4, time const.

$$t_s = \frac{4}{\xi\omega_n}$$

for 5% band it takes 3, time const.

$$t_s = 3/\xi\omega_n$$

These formula are not valid for over-damped.

for over-damped

$$t_s = \frac{2\xi}{\omega_n} \text{ for } 2\% \text{ band}$$

* Q. GATE 2002

The T/F fn. of a system is

$$\frac{G(s)}{} = \frac{100}{(s+1)(s+100)}$$

for a unit step i/p to system the approx. settling time for 2% criteria is

a) 100 sec.

b) 4 sec. ✓

c) 1 sec. (+)

D) 0.01 sec.